(1)

① $\int 9\sin^3 x \cdot \cos^2 x\, dx.$

$9\int \sin^3 x \cdot \cos^2 x\, dx.$

$9\int (1-\cos^2 x)\sin x \cdot \cos^2 x \cdot dx.$

$u = \cos x.$

$9\int -u^2 \cdot (1-u^2)\, du.$

$9\int -u^2 + u^4\, du.$

$= 9\int -u^2 + u^4\, du = 9\left(\frac{-u^3}{3} + \frac{u^5}{5}\right) + C$

$= 9\left(\frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5}\right) + C$

② $\int -8\sin^6 x \cos^3 x\, dx.$

$= -8\int (1-\sin^2 x)\cos x \cdot \sin^6 x\, dx$ put $u = \sin x.$

$= -8\int u^6 (1-u^2)\, du.$

$= -8\left(\frac{u^7}{7} - \frac{u^9}{9}\right) = -8\left(\frac{\sin^7(x)}{7} - \frac{\sin^9 x}{9}\right) + C.$

③ $\int 3\sec^2 x \tan x\, dx.$

$u = \tan x. \quad du = \sec^2 x \cdot dx.$

$= 3\int u\, du. = 3 \cdot \frac{u^2}{2}$

$= \frac{3}{2}\tan^2 x + C$

④ $5\int \tan^4 x\, dx$.

$$= 5\left(\frac{\tan^3 x}{3} - \left(\int \tan^2(x)\, dx\right)\right)$$

$$= \int \tan^2 x = -x + \tan x.$$

$$= 5\left(\frac{\tan^3 x}{3} - (-x + \tan x)\right)$$

$$= 5\left(\frac{1}{3}\tan^3 x + x - \tan x\right)$$

$$= 5\left(\frac{1}{3}\tan^3 x + x - \tan x\right) + c.$$

⑤ $\int -6\tan^3(2x) \cdot \sec^5(2x)\, dx.$

$$= -6\int \tan^3(2x)\sec^5(2x)\, dx.$$

$$= -6\int\left(-1 + \sec^2(2x)\right) \cdot \tan 2x \cdot \sec^5(2x)\, dx.$$

$$u = \sec(2x)$$

$$= -6\int \frac{u^4(u^2-1)}{2}\, du.$$

$$= -6\int \frac{u^6}{2} - \frac{u^4}{2}\, du.$$

$$= -6\left(\frac{u^7}{14} - \frac{u^5}{10}\right) = -6\,\frac{\sec^7(2x)}{14} - \frac{\sec^5(2x)}{10}.$$

$$= -6\left(\frac{\sec^7(2x)}{14} - \frac{\sec^5(2x)}{10}\right) + c.$$

(6) $\int -5\tan^5 x \cdot dx$. (3)

$$= -5\int (-1+\sec^2 x)^2 \tan x\,dx$$

$$= -5\int \frac{(-1+u^2)^2}{u}\,du$$

$$= -5\left(\int \frac{1}{u}\,du - \int 2u\,du + \int u^3\,du\right)$$

$$= -5\left(\ln|u| - u^2 + \frac{u^4}{4}\right)$$

$$= -5\left(\ln|\sec x| - \sec^2 x + \frac{\sec^4 x}{4}\right)$$

$$= -5\left(\ln|\sec x| - \sec^2 x + \frac{\sec^4 x}{4}\right) + C$$

(7). $$\int_{\pi/2}^{3\pi/4} 7\sin^5 x\cdot\cos^3 x\,dx = 7\int_{\pi/2}^{3\pi/4} \sin^5 x\cdot\cos^3 x\,dx$$

$$= 7\int_{\pi/2}^{3\pi/4} \sin^5 x\cdot\cos^3 x\,dx$$

$u = \cos x$.

$$7\int_{\pi/2}^{3\pi/4} \sin^5 x\cdot\cos^3 x\,dx = 7\int_0^{-\sqrt{2}/2} -u^3(1-u^2)^2\,du$$

$$= 7\int_0^{-\sqrt{2}/2} -u^3+2u^5-u^7\,du$$

$$= 7\left(-\frac{1}{16} - \frac{1}{24} + \frac{1}{128}\right) = \frac{-77}{384}$$

(8) $\int_0^{\pi/2} 7\cos^2 x\, dx.$

$$\frac{7}{2}\int_0^{\pi/2} 1+\cos(2x)\,dx. = \frac{7}{2}\left(\int_0^{\pi/2} dx + \int_0^{\pi/2} \cos(2x)\,dx.\right.$$

$$= \frac{7}{2}\left(\frac{\pi}{2}+0\right) = \underline{\underline{\frac{7\pi}{4}}}$$

(9) $\int_0^{\pi/2} -5\sin^2(3x)\,dx. \quad = -5\,\frac{1}{2}\int_0^{\pi/2} 1-\cos(6x)\,dx.$

$$= \cancel{}\left(\cancel{}\right) \quad \frac{-5}{2}\left(\frac{\pi}{2}-0\right)$$

$$= \underline{\underline{\frac{-5\pi}{4}}}$$

(10) $\int_0^{\pi/4} -9\sin^4 x\cdot\cos^2 x\, dx.$

put $u = \sin x.$

$$= -9\int_0^{\pi/4} \sin^4 x\,(1-\sin^2 x)\,dx.$$

$$= -9\int_0^{\pi/4} \sin^4 x - \sin^6 x\, dx.$$

$$= -9\left[\int_0^{\pi/4} \sin^4 x - \int_0^{\pi/4} \sin^6 x\, dx\right] \quad \text{---} (1)$$

$$\int_0^{\pi/4} \sin^4 x = -\left[\frac{\cos x\cdot\sin^3 x}{4}\right]_0^{\pi/4} + \frac{3}{4}\int_0^{\pi/4} \sin^2 x\, dx.$$

$$= -\frac{\cos x\cdot\sin^3 x}{4}\Bigg|_0^{\pi/4} + \frac{3}{4}\left(\frac{\pi-2}{8}\right)$$

$$= -\left[\frac{1}{4}\sin^3 x\cdot\cos x\right]_0^{\pi/4} + \frac{3(\pi-2)}{32}.$$

$$= \underline{\underline{\frac{-1}{16} + \frac{3(\pi-2)}{32}}} \quad \text{---} (2).$$

$$\int_0^{\pi/4} \sin^6 x\,dx = -\left[\frac{\cos x \cdot \sin^5 x}{6}\right]_0^{\pi/4} + \frac{5}{6}\int_0^{\pi/4} \sin^4 x\,dx.$$ (5)

$$= -\frac{1}{6}\sin^5 x \cos x \Big|_0^{\pi/4} + \frac{5(3\pi-8)}{192}$$

$$= \frac{-1}{48} + \frac{5(3\pi-8)}{192}$$

Substituting these values,

$$= 9\left(\frac{-1}{16} + \frac{3(\pi-2)}{32} - \left(\frac{-1}{48} + \frac{5(3\pi-8)}{192}\right)\right)$$

$$= -9\left(\frac{-1}{24} + \frac{3\pi+4}{192}\right)$$

(12)

$$10\int_0^{\pi/4} \sec^4 x \cdot \tan^4 x\,dx.$$

$$= 10\int_0^{\pi/4} (1+\tan^2 x)\sec^2 x \tan^4 x\,dx.$$

$u = \tan x. \quad du = \sec^2 x\,dx.$

$$= 10\int_0^1 u^4(1+u^2)\,du = 10\int_0^1 u^4(1+u^2)\,du.$$

$$= 10\int_0^1 u^4 + u^6\,du.$$

$$= 10\left(\frac{1}{5} + \frac{1}{7}\right) = \frac{24}{7}$$

(12). $\int_0^{\pi/2} 9\sin^2(x).\cos^2 x\,dx.$

$9\int_0^{\pi/2}\frac{1-\cos 4x}{8} = \frac{9}{8}\left(\int_0^{\pi/2} 1\,dx - \int_0^{\pi/2}\cos(4x)\,dx.\right.$

$= \frac{9}{8}\left(\frac{\pi}{2}-0\right) = \underline{\underline{\frac{9\pi}{16}}}$

$\int_0^{\pi/2}\cos(4x)$

$u=4x.$

$\frac{1}{4}\int_0^{2\pi}\cos u\,du = \frac{1}{4}\sin u\Big|_0^{2\pi} = \frac{1}{4}\times 0 = \underline{\underline{0}}$

$\therefore \int_0^{\pi/2} 9\sin^2 x.\cos^2 x\,dx = \underline{\underline{\frac{9\pi}{16}}}.$

(13). $\int_0^{\pi/3} 10\tan^5 x.\sec^4 x\,dx.$

$= 10\int_0^{\pi/3}(1+\tan^2 x)\sec^2 x\tan^5 x\,dx.$

$u=\tan x \qquad du=\sec^2 x\,dx.$

$= 10\int_0^{\sqrt{3}} u^5(1+u^2)du.$

$= 10\int_0^{\sqrt{3}}(u^5+u^7)\,du = 10\left(\frac{9}{2}+\frac{81}{8}\right)$

$= \underline{\underline{\frac{585}{4}}}.$